Bernoulli Trials

Tuesday, February 28, 2012
3:12 PM

Bernoulli Trials $\rightarrow c(t)$

	$\cdots$ X_{-3}	X_{-2}	X_{-1}	X_0	X_1	X_2	X_3	$\cdots$
Coin flipping sequence:	$\cdots$ T	H	T	H	T	T	H	$\cdots$
Bernoulli sequence:	$\cdots$ 0	1	0	1	0	0	1	$\cdots$
Binary independent random sequence:	$\cdots$ 1	−1	1	−1	1	1	−1	$\cdots$
	$\cdots$ A	−A	A	−A	A	A	−A	$\cdots$

Disadvantage: Random $\rightarrow$ Require large storage @ Tx, Rx

Advantages: ① Balanced property

$\{0,1\}$-version : fraction of 1 $\approx \frac{1}{2}$

fraction of 0 $\approx \frac{1}{2}$

② Run-length property

Run: consecutive 0s or consecutive 1s

For example,

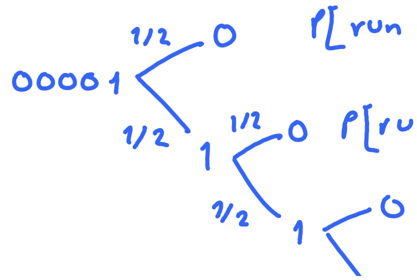
a run (of 1s) of length 5

010001000 1011111 000

a run (of 1) of length 1

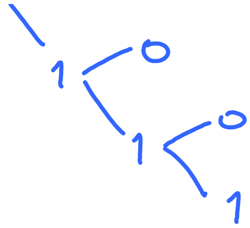
a run (of 0s) of length 3

Back to Bernoulli sequence



$P[\text{run length} = 1] = \frac{1}{2}$

$P[\text{run length} = 2] = \frac{1}{2^2}$



$P[\text{run length} = \ell] = \frac{1}{2^\ell} \leftarrow$ geometric pmf.

③ shift property

use $\{-1,1\}$-version

$\cdots \quad x_{-2} \quad x_{-1} \quad x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \cdots$

shifted version $\quad \cdots \quad \underbrace{x_0 \quad x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6}_{n \text{ of these}} \cdots$

Normalized correlation: $\frac{1}{7} \sum_{i=-2}^{4} \underbrace{x_i x_{i+2}}$ $\approx \frac{\text{small}}{7}$

↳ half: same → 1

↳ half: different → −1

$$\frac{1}{n} \sum_{i=i_0}^{i_0+n-1} x_i x_{i+m} \xrightarrow[\text{LLN}]{} \mathbb{E}[x_i x_{i+m}]$$

independent $\quad = \mathbb{E}[x_i]\,\mathbb{E}[x_{i+m}]$

$\{1,-1\} \quad = 0 \times 0 = 0$

$m \neq 0$

$m = 0 \qquad \to 1$